Rounding and Percent Error

ROUNDING OFF TO SIGNIFICANT FIGURES

To round off to n significant figures, we look at the $(n+1)$th digit.

- If it is 0, 1, 2, 3 or 4 we do not change the nth digit.
- If it is 5, 6, 7, 8 or 9 we increase the nth digit by 1.

We delete all digits after the nth digit, replacing by 0s if necessary.

Example 13 — Self Tutor

Round: **a** 7.182 to 2 significant figures **b** 0.001 32 to 2 significant figures **c** 423 to 1 significant figure **d** 4.057 to 3 significant figures.

a $7.182 \approx 7.2$ (2 s.f.)

This is the 2nd significant figure, so we look at the next digit which is 8. The 8 tells us to round the 1 up to a 2 and leave off the remaining digits.

b $0.001\,32 \approx 0.0013$ (2 s.f.)

These zeros at the front are place holders and so must stay. The first significant figure is the 1. The third significant figure, 2, tells us to leave the 3 as it is and leave off the remaining digits.

2 Write correct to 3 significant figures:

a 43 620 **b** 10 076 **c** $0.\overline{6}$ **d** 0.036 821 **e** 0.318 6
f 0.719 6 **g** $0.\overline{63}$ **h** 0.063 71 **i** 18.997 **j** 256 800

3 Write correct to 4 significant figures:

a 28.039 2 **b** 0.005 362 **c** 23 683.9 **d** 42 366 709
e 0.038 792 **f** 0.006 377 9 **g** 0.000 899 9 **h** 43.076 321

Percentage error $E = \dfrac{|V_A - V_E|}{V_E} \times 100\%$

Example 20 — Self Tutor

You estimate a fence's length to be 70 m whereas its true length is 78.3 m. Find, correct to one decimal place:

a the error **b** the percentage error.

a $\text{error} = V_A - V_E$
$= 70 - 78.3$
$= -8.3$ m

b percentage error
$= \dfrac{|V_A - V_E|}{V_E} \times 100\%$
$= \dfrac{|-8.3|}{78.3} \times 100\%$
$\approx 10.6\%$

1 Find **i** the error **ii** the percentage error in rounding:

a the yearly profit of €1 367 540 made by a company to €1.37 million
b a population of 31 467 people to 31 000 people
c a retail sales figure of \$458 110 to \$460 000
d the number of new cars sold by a company in a year from 2811 to 3000.

2 Find **i** the error **ii** the percentage error if you estimate:

a the mass of a brick to be 5 kg when its actual mass is 6.238 kg
b the perimeter of a property to be 100 m when its actual length is 97.6 m
c the capacity of a container to be 20 L when its actual capacity is 23.8 L
d the time to write a computer program to be 50 hours when it actually takes 72 hours.

Linear Equations

Two unknowns mean two equations. Write two equations and solve using PlySmlt.

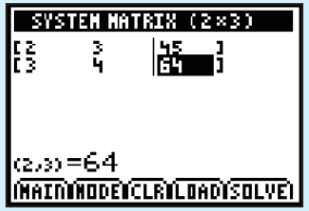
Example 25 — Self Tutor

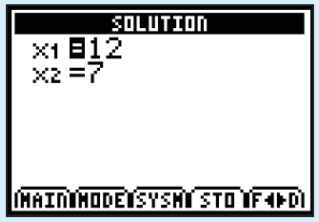
Two adults' tickets and three children's tickets to a baseball match cost \$45, while three adults' and four children's tickets cost \$64. Find the cost of each type of ticket.

Let \$$x$ be the cost of an adult's ticket and \$$y$ be the cost of a child's ticket.

So, $2x + 3y = 45$ and $3x + 4y = 64$

TI-84 Plus





The solution is $x = 12,\ y = 7$.

So, an adult's ticket costs \$12 and a child's ticket costs \$7.

3 A hairdresser has 13 small and 14 large cans of hairspray, giving a total of 9 L of hairspray. At this time last year she had 4 small and 12 large cans, totalling 6 L of hairspray. How much spray is in each size can?

4 A violinist is learning a waltz and a sonatina. One day she practices for 33 minutes by playing the waltz 4 times and the sonatina 3 times. The next day she plays the waltz 6 times and the sonatina only once, for a total of 25 minutes. Determine the length of each piece.

Quadratics

Factorised Form $\boldsymbol{y = a(x - \alpha)(x - \beta)}$

α *and* β are the x-intercepts

Y-intercept = $a(\alpha)(\beta)$

Vertex: x=$\frac{\alpha+\beta}{2}$, find y by substituting x into f(x)

$\boldsymbol{y = ax^2 + bx + c}$ Form

y-intercept is the c

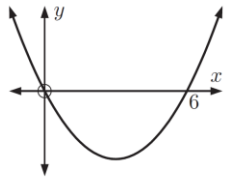
x-intercept are the factors, PlySmlt

vertex and axis of symmetry: $x = -\frac{b}{2a}$

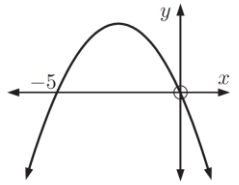
Vertex is the max or min of the graph.

For each of the following, find the equation of the axis of symmetry:

a



b



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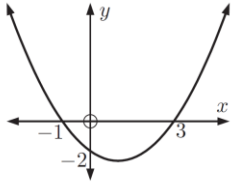


For each of the following functions:

i find the axes intercepts

ii find the equation of the axis of symmetry

iii find the coordinates of the vertex

iv sketch the function, showing all important features

v state the domain and range of the function.

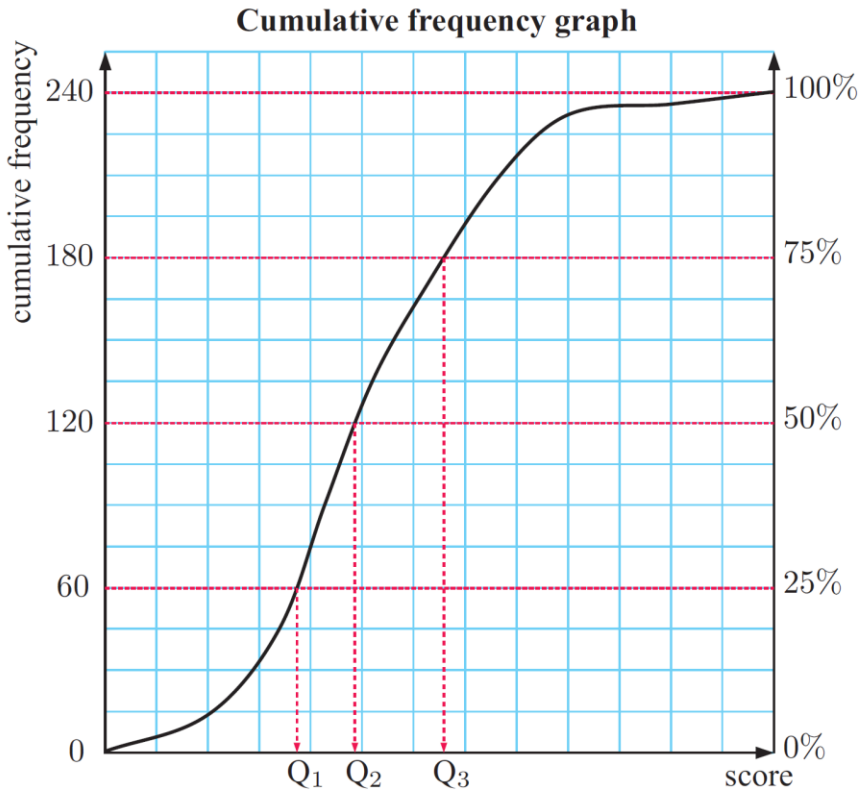
a $y = x^2 - 4x + 3$

b $y = -(x + 2)(x - 6)$

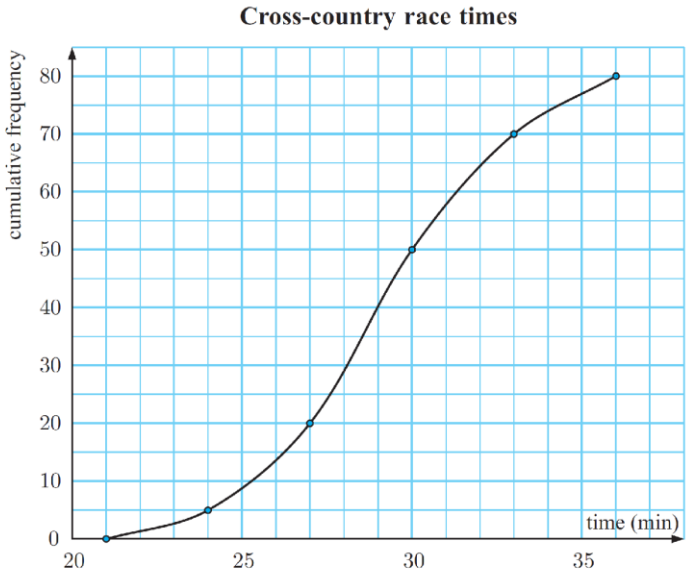
Cumulative Frequency Graphs

A **percentile** is the score below which a certain percentage of the data lies.

- the 85th percentile is the score below which 85% of the data lies.
- If your score in a test is the 95th percentile, then 95% of the class have scored less than you.



5 The following cumulative frequency graph displays the performance of 80 competitors in a cross-country race.



Find:

a the lower quartile time

b the median

c the upper quartile

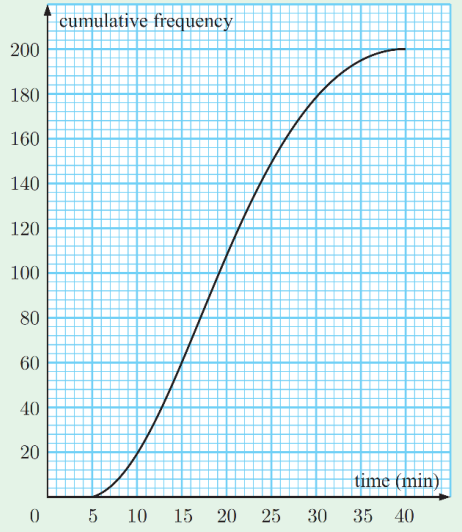
d the interquartile range

e an estimate of the 40th percentile.

5 This cumulative frequency curve shows the times taken for 200 students to travel to school by bus.

a Estimate how many of the students spent between 10 and 20 minutes travelling to school.

b 30% of the students spent more than m minutes travelling to school. Estimate the value of m.

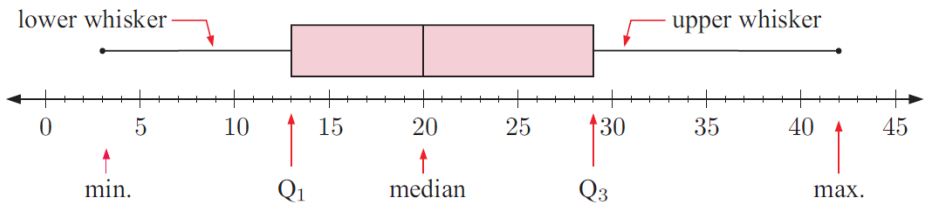


Box and Whisker Plots

- the minimum value
- the lower quartile (Q_1)
- the median (Q_2)
- the upper quartile (Q_3)
- the maximum value

These five numbers form the **five-number summary** of the data set.

minimum $= 3$
$Q_1 = 13$
median $= 20$
$Q_3 = 29$
maximum $= 42$



8 Consider this set of data:

19, 7, 22, 15, 14, 10, 8, 28, 14, 18, 31, 13, 18, 19, 11, 3, 15, 16, 19, 14

a Find the 5-number summary for the data. **b** Find the range and IQR of the data.

c Draw a boxplot of the data set.

33 The ages in months of 20 students are:

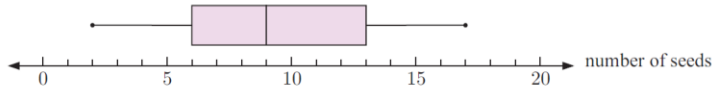
198, 192, 195, 194, 205, 208, 210, 200, 206, 203,
196, 198, 196, 201, 194, 198, 197, 195, 209, 204.

a Find the:

i median **ii** range **iii** interquartile range of the data.

b Draw a box and whisker plot for the ages of the students.

108 Margaret picked some mandarins from a tree, and counted the number of seeds in each. Her results are shown in the boxplot below.



Find the:

a median **b** interquartile range **c** range of the data.

Calculus

Derivatives find gradients.
Rewrite denominators as numerators.
Tangents

1. Find y by using f(x). Need (x, y)
2. Find f'(x).
3. Find m using f'(x)
4. Find b using (x, y), m and y=mx+b

Normals
Same as above but switch m to its opposite reciprocal and continue.

Find the equation of the tangent to:

a $y = x^2$ at $x = 4$

b $y = x^3$ at $x = -2$

c $y = 3x^{-1}$ at $x = -1$

d $y = \frac{4}{x^3}$ at $x = 2$

Optimisation

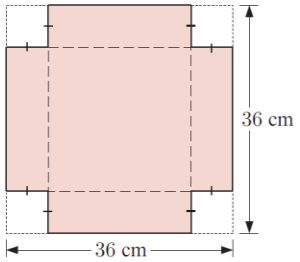
Steps

1. Create a formula to be optimized– single term(x)
   Write any restrictions
2. Take 1st derivative = zero

10 Sam has a sheet of metal which is 36 cm by 36 cm square. He will cut out identical squares which are x cm by x cm from the corners of the sheet. He will then bend the sheet along the dashed lines to form an open container.

a Show that the capacity of the container is given by $V(x) = x(36 - 2x)^2$ cm^3.

b What sized squares should be cut out to produce the container of greatest capacity?

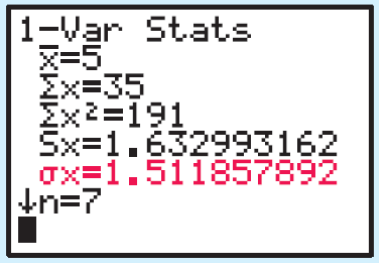


Mean, Median and Standard Deviation

Example 18

Calculate the standard deviation of the data set: 2, 5, 4, 6, 7, 5, 6.

TI-84 Plus



3 The weights of a group of cooking chickens in kilograms are:

1.5, 1.8, 1.7, 1.4, 1.7, 1.8, 2.0, 1.5, 1.6, 1.6, 1.9, 1.7, 1.4, 1.7, 1.8, 2.0

Use technology to find the mean and standard deviation of weights.

5 The weights of ten young turkeys to the nearest 0.1 kg are:
0.8, 1.1, 1.2, 0.9, 1.2, 1.2, 0.9, 0.7, 1.0, 1.1

a Find the mean and standard deviation for the weights of the turkeys.

b After being fed a special diet for one month, the weights of the turkeys doubled. Find the new mean and standard deviation.

c Comment on your results.

STANDARD DEVIATION FOR GROUPED DATA

For **continuous** data, or data that has been grouped in **classes**, we use the **mid-interval values** to represent all data in that interval.

GRAPHICS CALCULATOR INSTRUCTIONS

Example 19 — Self Tutor

Use technology to estimate the standard deviation for this distribution of examination scores:

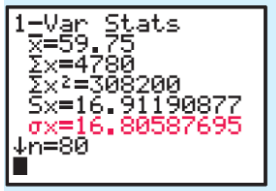
Mark	*Frequency*	*Mark*	*Frequency*
0 - 9	1	50 - 59	16
10 - 19	1	60 - 69	24
20 - 29	2	70 - 79	13
30 - 39	4	80 - 89	6
40 - 49	11	90 - 99	2

In order to estimate the standard deviation of already grouped data, the mid-interval values are used to represent all data in that interval.

We then use technology to estimate the standard deviation.

Class interval	*Mid-interval value*	*Frequency*	*Class interval*	*Mid-interval value*	*Frequency*
0 - 9	4.5	1	50 - 59	54.5	16
10 - 19	14.5	1	60 - 69	64.5	24
20 - 29	24.5	2	70 - 79	74.5	13
30 - 39	34.5	4	80 - 89	84.5	6
40 - 49	44.5	11	90 - 99	94.5	2

TI-84 Plus



5 The weekly wages (in dollars) of 200 steel workers are given alongside.
Estimate the mean and the standard deviation of the data.

Wage ($)	*Number of workers*
360 - 369.99	17
370 - 379.99	38
380 - 389.99	47
390 - 399.99	57
400 - 409.99	18
410 - 419.99	10
420 - 429.99	10
430 - 439.99	3

4 The lengths of 30 randomly selected 12-day old babies were measured and the following data obtained:

Length (cm)	[40, 42)	[42, 44)	[44, 46)	[46, 48)	[48, 50)	[50, 52)	[52, 54)
Frequency	1	1	3	7	11	5	2

Estimate the mean length and the standard deviation of the lengths.

Normal Distribution

$X \sim N(\mu, \sigma^2)$ means normal distribution with a mean of μ
And standard deviation of σ.

Example 2 Self Tutor

If $X \sim \text{N}(10,\ 2.3^2)$, find these probabilities:

a $\text{P}(8 \leqslant X \leqslant 11)$ **b** $\text{P}(X \leqslant 12)$ **c** $\text{P}(X > 9)$. Illustrate your results.

b Using a **TI-84 Plus**:

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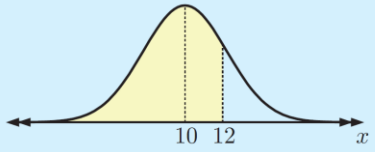
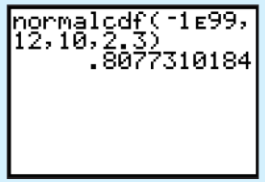




$\therefore\ \text{P}(X \leqslant 12) \approx 0.808$

X is a random variable that is distributed normally with mean 32 and standard deviation 6. Find:

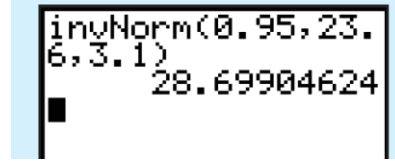
a $\text{P}(25 \leqslant X \leqslant 30)$ **b** $\text{P}(X > 27)$ **c** $\text{P}(22 \leqslant X \leqslant 28)$

d $\text{P}(X \leqslant 30.9)$ **e** $\text{P}(X < 23.8)$ **f** $\text{P}(22.1 < X < 32.1)$

Example 4

If $X \sim \text{N}(23.6,\ 3.1^2)$, find k for which $\text{P}(X < k) = 0.95$.

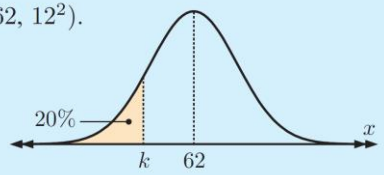
TI-84 Plus



Let X denote the final examination result, so $X \sim \text{N}(62,\ 12^2)$.

We need to find k such that $\text{P}(X \geqslant k) = 0.8$

$\therefore\ \text{P}(X \leqslant k) = 0.2$



4 Given that $X \sim \text{N}(23,\ 5^2)$, find a such that:

a $\text{P}(X < a) = 0.378$ **b** $\text{P}(X \geqslant a) = 0.592$ **c** $\text{P}(23 - a < X < 23 + a) = 0.427$

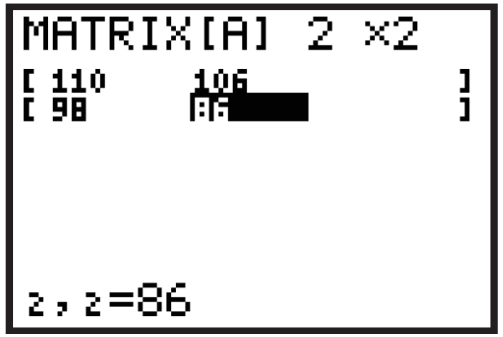
THE χ^2 TEST OF INDEPENDENCE

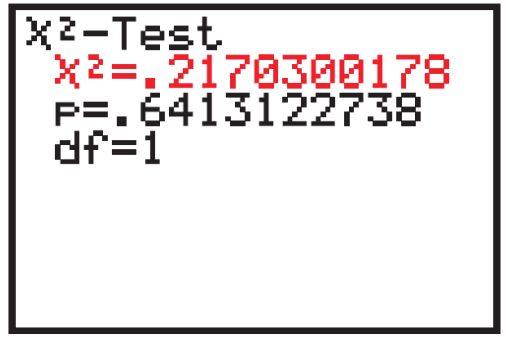
The hair and eye colours of 150 randomly selected individuals are shown in the table below.

Eye colour \ Hair colour	Blond	Black	Brunette	Red
Blue	14	10	21	5
Brown	11	32	20	12
Green	5	2	14	4

At a 5% significance level, the critical value for χ^2 is 12.59.
Test, at a 5% level, whether there is an association between *hair colour* and *eye colour*.

1. Write the null hypothesis.
2. Write the alternative hypothesis.
3. Find the χ^2 calculated value.
4. Do you reject or not reject the null hypothesis?





Step 1: State H_0 called the **null hypothesis**. This is a statement that the two variables being considered are independent.
State H_1 called the **alternative hypothesis**. This is a statement that the two variables being considered are not independent.

Step 2: State the **rejection inequality** $\chi^2_{calc} > k$ where k is the **critical value** of χ^2.

Step 3: Construct the expected frequency table.

Step 4: Use technology to find χ^2_{calc}.

Step 5: We either reject H_0 or do not reject H_0, depending on the result of the rejection inequality.

Step 6: We could also use a ***p*-value** to help us with our decision making.
For example, at a 5% significance level: If $p < 0.05$, we reject H_0.
If $p > 0.05$, we do not reject H_0.

Currency Conversions

Example 25 — Self Tutor

A banker changes South African rand to other currencies at a fixed commission of 1.5%. Wendy wishes to convert 800 ZAR to Russian rubles where 1 ZAR buys 3.75 RUB.

a What commission is charged? **b** How much does Wendy receive?

a Commission $= 800 \text{ ZAR} \times 1.5\%$
$= 800 \times 0.015 \text{ ZAR}$
$= 12 \text{ ZAR}$

b Wendy receives 788×3.75 rubles
$= 2955$ rubles

Always find the commission and subtract it off before you convert to another currency.

1 A bank exchanges GBP for a commission of 1.5%. For the following transactions, calculate:

i the commission charged **ii** how much the customer receives.

a 500 GBP is converted to US dollars where 1 GBP buys 1.5616 USD.

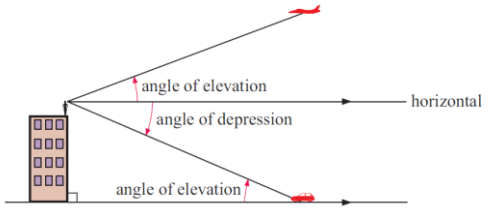
b 350 GBP is converted to euros where 1 GBP buys €1.1605.

c 1200 GBP is converted to New Zealand dollars where 1 GBP buys $2.0954 NZ.

Right Triangle Trigonometry

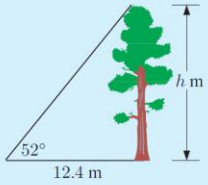
ANGLES OF ELEVATION AND DEPRESSION

The angle between the horizontal and your line of sight to an object is called the **angle of elevation** if you are looking upwards, or the **angle of depression** if you are looking downwards.



Example 9 — Self Tutor

When measured from a point 12.4 m from its base, the angle of elevation to the top of a tree is 52°. Find the height of the tree.



Let h be the tree's height in metres.

For the 52° angle, OPP $= h$ and ADJ $= 12.4$

$\therefore \tan 52° = \frac{h}{12.4}$

$\therefore 12.4 \times \tan 52° = h$

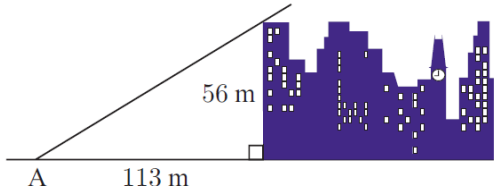
$\therefore h \approx 15.9$

So, the tree is 15.9 m high.

1 When measured from a point 9.32 m from its base, the angle of elevation to the top of a flagpole is 63°. Find the height of the flagpole.

5 **a** Find the angle of elevation to the top of a 56 m high building from point A, which is 113 m from its base.

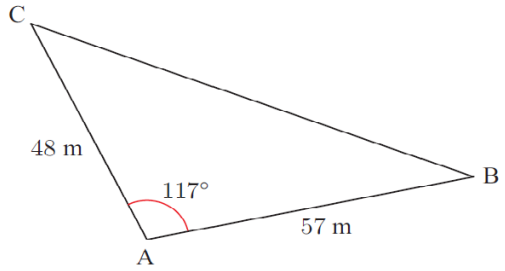
b What is the angle of depression from the top of the building to A?



Non-right Triangle Trigonometry Lesson 15I

c Find the coordinates of the vertex of the graph.

46 The diagram shows the plan of a triangular garden bed. The garden bed will be enclosed by a 50 cm high wall and then filled with soil.

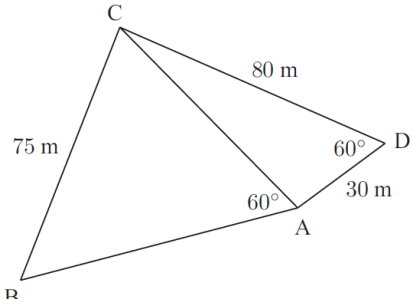


a Calculate the length BC.

b Calculate the area of the garden bed.

c Find the volume of soil needed to fill the garden bed.

103



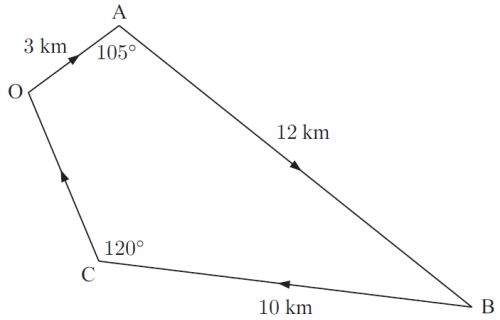
The figure shows two adjacent triangular fields ABC and ACD. AD = 30 m, CD = 80 m, BC = 75 m, $A\hat{D}C = 60°$, and $B\hat{A}C = 60°$.

a Calculate the length of AC.

b Calculate the size of $A\hat{B}C$.

c Find the total area of the fields.

52 A yachting course is illustrated in the diagram alongside. The yachts start and finish at O, and travel in the direction indicated.



a Find the distance from O to B in a straight line.

b Find $B\hat{O}C$.

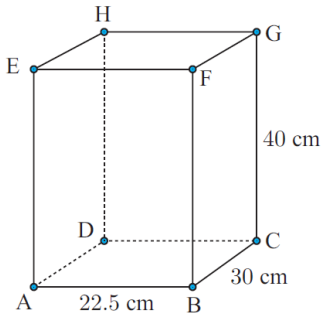
c Find the length of OC.

d Calculate the area enclosed by the course OABC.

e The course designer stated the length of the course is 30 km. Calculate the percentage error in this approximation.

3-D Trigonometry Lesson 15D

39 The diagram shows a cuboid which measures 22.5 cm by 30 cm by 40 cm.



a Find the length of AC.

b Find the area of the plane ACGE.

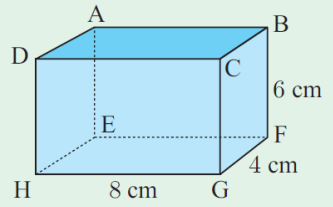
c Find the volume of the triangular prism ACGEFB.

d Find the length of CE.

e Find $A\hat{C}E$.

f Let M be the midpoint of CE. Find the area of triangle AMC.

5 For the rectangular prism shown, find the angle that:



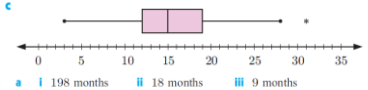
a AH makes with HG

b DF makes with the base plane EFGH.

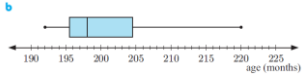
Answers

Box and Whisker Plots

8 a min = 3; $Q_1 = 12$; med = 15; $Q_3 = 19$; max = 31
b range = 28; IQR = 7
c



33 a i 198 months ii 18 months iii 9 months
b

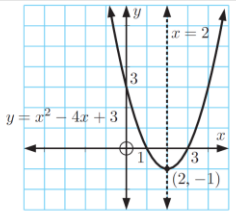


108 a 9 seeds b 7 seeds c 15 seeds

Quadratic Functions

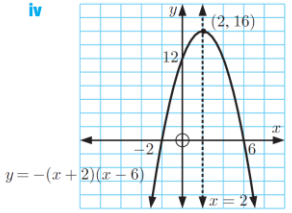
1 a $x = 3$ b $x = -\frac{5}{2}$ c $x = 1$ d $x = -4$

3 a i x-intercepts: 1 and 3
y-intercept: 3
ii $x = 2$
iii $(2, -1)$
iv



v Domain $= \{x \mid x \in \mathbb{R}\}$, Range $= \{y \mid y \geqslant -1\}$

b i x-intercepts: -2 and 6
y-intercept: 12
ii $x = 2$
iii $(2, 16)$
iv



v Domain $= \{x \mid x \in \mathbb{R}\}$, Range $= \{y \mid y \leqslant 16\}$

Currency Conversions

1 a i 7.50 GBP ii 769.09 USD
b i 5.25 GBP ii €400.08
c i 18 GBP ii $2476.76 NZ

Rounding and Percent Error

2 a 43 600 b 10 100 c 0.667 d 0.0368 e 0.319
f 0.720 g 0.636 h 0.0637 i 19.0 j 257 000

3 a 28.04 b 0.005 362 c 23 680 d 42 370 000
e 0.038 79 f 0.006 378 g 0.000 899 9 h 43.08

1 a i €2460 ii 0.180% b i −467 people ii 1.48%
c i $1890 ii 0.413% d i 189 cars ii 6.72%

2 a i −1.238 kg ii 19.8% b i 2.4 m ii 2.46%
c i −3.8 L ii 16.0% d i −22 hours ii 30.6%

Cumulative Frequency

5 a 27 min b 29 min c 31.3 min
d 4.3 min e ≈ 28 min

5 a 88 students b $m = 24$

$\chi^2 - Test$

1. Eye color is independent of hair color.
2. Eye color is not independent of hair color.
3. $\chi^2 = 18.4$
4. We reject the null hypothesis. Eye color is not independent of hair color.

Right Triangle Trigonometry

1. 18.3 m 5. a. 26.4° b. 26.4°

Non-right Triangle Trigonometry

46 a 89.7 m b 1220 m² c 609 m³

103 a 70 m b 53.9° c 3440 m²

52 a 13.1 km b 41.4° c 4.83 km d 38.3 km²
e 0.567%

Normal Distribution

3 a 0.248 b 0.798 c 0.205 d 0.427
e 0.0859 f 0.457

4 a 21.4 b 21.8 c 2.82

Calculus

1 a $y = 8x - 16$ b $y = 12x + 16$ c $y = -3x - 6$
d $y = -\frac{3}{4}x + 2$ e $y = 7x - 5$ f $y = -3x - 5$

10. 6 cm by 6 cm

Linear Equations

3 Small can = 240 mL, Large can = 420 mL

4 Waltz = 3 minutes, Sonatina = 7 minutes

Mean, Median and Standard Deviation

3 mean ≈ 1.69 kg, standard deviation ≈ 0.182 kg

5 a $\overline{x} = 1.01$ kg; $s = 0.17$ b $\overline{x} = 2.02$ kg; $s = 0.34$
c Doubling the values doubles the mean and standard deviation.

4 $\overline{x} \approx 48.3$ cm, $s_n \approx 2.66$ cm 5 $\overline{x} \approx \$390.30$, $s_n \approx \$15.87$

3-D Trigonometry

39 a 37.5 cm b 1500 cm² c 13 500 cm³
d 54.8 cm e 46.8° f 375 cm²

5 a 90° b 33.9°